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On-Line Sensor Monitoring in an Active Front Steering System using Extended Kalman Filtering

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ABSTRACT

This contribution describes several ways of realising application dependent safety measures for the motor angle sensor of the active front steering system. Observer based methods, based on mathematical models of the motor under investigation are derived, realised and tested in a prototype vehicle. A comparison is carried out with respect to accuracy, tracking and computational load.

INTRODUCTION AND MOTIVATION

Active front steering is a newly developed technology for passenger cars that realises an electronically controlled superposition of an angle to the hand steering wheel angle that is prescribed by the driver.

A great deal of functionality that is housed in the electronic control unit is devoted to ensure the overall functional safety of the system. This paper will focus on safety measures that are needed to reach the integrity of the sensor, measuring the position of the electric motor. Beyond usual sensor diagnosis such as analogue signal monitoring, test patterns etc (all of them described in recognised safety standards such as [2]), more advanced methods are needed in order to detect failures of certain types or in certain situations.

One method used within active front steering is position estimation using filtering. Therefore, a motor model, along with validated parameters has been established. Since motor speed is one state, it can be estimated using observer techniques (measuring voltages and currents of the motor). Several techniques, linear and nonlinear ones, such as a full state Luenberger observer, a reduced state Luenberger observer and an extended Kalman filter are presented and compared. Computational burden and applicability for production type electronic control units is discussed as well.

Outline of the paper First, basic system description and notation is established, followed by a short background on functional safety. Models for the motor to be monitored are compiled then, after that, several approaches for state estimation are described. Results with measurement data from a vehicle equipped with active front steering are presented and discussed thereafter.

SYSTEM DESCRIPTION AND NOTATION

The complete system setup including mathematical modelling and parameter estimation is described in great detail in [4]. In order to make this paper self-contained, the basic relations are given here as well. Fig. 1 shows the active front steering principle: The driver controls the vehicles

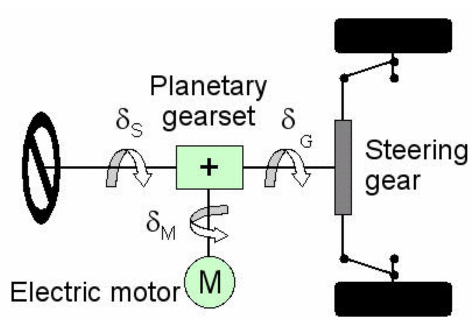

Figure 1: Schematic view of active front steering system. The electric motor superimposes an angle δ_M to the hand steering wheel angle δ_S. The result is the steering gear's pinion angle δ_G.

course via the hand steering wheel; the resulting steering wheel angle is denoted by δ_S. Active front steering actuates an additional angle δ_M using its electric motor. Both angles result in an pinion angle δ_G down at the steering rack. All three angles relate as given in (1), also accounting for the respective ratios i_M, i_D. The resulting (average) road wheel angle can then be calculated via the pinion angle and a static nonlinearity $F_{SG}(\cdot)$ that accounts for the relation between pinion angle and rack displacement as well as for the steering geometry, cf. (2). Finally, the overall ratio between hand wheel to front road wheel $\delta_F(t)$ is defined in (3).

$$\delta_G(t) = \frac{1}{i_M} \cdot \delta_M(t) + \frac{1}{i_D} \cdot \delta_S(t) \quad (1)$$

$$\delta_G(t) = F_{SG}\left(\delta_F(t)\right) \quad (2)$$

$$\delta_F(t) = \frac{1}{i_V} \cdot \delta_S(t) \quad (3)$$

Having this basic framework at hand, one can start looking at functions that manipulate the motor angle $\delta_M(t)$ in order to e.g. achieve a desired overall steering ratio i_V that depends on vehicle speed and pinion angle, i.e.:

$$i_V = i_V(v_X, \delta_G) \quad (4)$$

This desired motor angle $\delta_{Md}(t)$ will then be passed to the motor's feedback control algorithm. However, before designing such functions, the plausibility of all signals discussed so far has to be ensured, to ensure the safety of the system. This is part of the Functional Safety, described next.

FUNCTIONAL SAFETY

The so-called technical safety concept deals with ensuring the functional safety of the system, which means that no harmful actions are initiated (with a prescribed probability). The analysis process described in [7] assigns a certain safety integrity level for each component in a top-down approach. Here, components can be actual devices such as sensors, microcontrollers, motors or functions (which are then mapped onto software modules). Having assigned a certain safety integrity level to a component, for example a sensor, a certain amount of safety measures has to be implemented until the risk reduction (as intended by the safety concept) is reached. Safety measures are usually classified as being *electrics/electronics dependent* or *application dependent*. The first category can be set up whenever the component in question is in place – in any system. Examples are simple range and gradient checks of voltages that generate a sensor signal, watchdogs for microcontrollers etc. In safety standards [2], the diagnostic coverage of these safety measures is low or medium, since they only represent necessary conditions for proper functionality. Hence, in systems of higher safety integrity level, they are accompanied by application dependent safety measures. These are based on application dependent relations. As an example, any of the sensors in the active front steering system could be validated exploiting (1) and assuming that the other two signals are valid. The information obtained from both types of safety functions is collected, the current state of the signals and the system is assessed in the Failure Diagnosis and Management System (FDMS).

This contribution is concerned with the application dependent safety measures of the motor angle sensor of the active front steering system. In general, application dependent safety measures share a generic structure that is depicted in Fig. 2 and already well established in literature [1, 9]. The signal or state $y(t)$, to be monitored, is compared to its estimate, generated for instance by a model also called filter. Most importantly, the filter has to use signals $u(t)$ that are independent of the signal to be estimated. The difference between signal and its estimate is called *residuum* $\epsilon(t)$. The distance measure then generates the *symptom* $s(t)$. Finally, the *stopping rule* decides whether or not to raise an alarm. Usual stopping rules are direct thresholding, generalised moving average, cumulated sums etc. Mean, variance and other statistical properties could be used as distance measures. We refer to [1] for a state of the art overview of such techniques. Criteria for chosing one method over another are the trade-off between mean time to detection and mean time to false alarm, but also computational load etc.

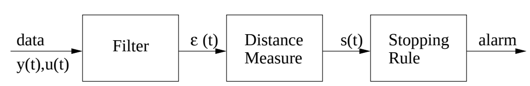

Figure 2: Generic structure of a safety measure containing filter, distance measure and stopping rule.

MODELS OF THE BLDC MOTOR

Fig. 3 shows the BLDC motor including the different co-ordinate transformations and the current controller, realised in rotor (d, q) co-ordinates. Phase currents I_U, I_V, I_W of the BLDC motor are measured. These phase currents are transformed to stator (α, β) co-ordinates using the so-

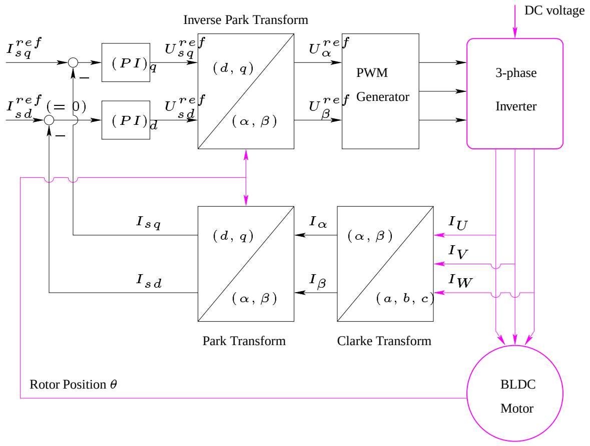

Figure 3: Park/Clarke Transforms and Generation of PWM Signals.

called *Clarke Transform*:

$$I_\alpha = I_U \qquad (5)$$

$$I_\beta = \frac{1}{\sqrt{3}} I_U + \frac{2}{\sqrt{3}} I_V \qquad (6)$$

$$0 = I_U + I_V + I_W. \qquad (7)$$

Note, that these co-ordinates are *time variant* ones. Aiming at an *time invariant* co-ordinate system, they are transformed to (d,q) co-ordinates using the so-called *Park Transform:*

$$I_{sd} = I_\alpha \cos(\theta) + I_\beta \sin(\theta) \qquad (8)$$

$$I_{sq} = -I_\alpha \sin(\theta) + I_\beta \cos(\theta). \qquad (9)$$

Note, however, that this transformation is depending on the rotor position θ. These co-ordinates are attached to the rotor flux and torque respectively. The two controllers (for torque and flux component) generate voltages $U_{sq}^{ref}, U_{sd}^{ref}$ that need to be transformed to stator-based co-ordinates using the *Inverse Park Transform*:

$$U_\alpha^{ref} = U_{sd}^{ref} \cos(\theta) - U_{sq}^{ref} \sin(\theta) \qquad (10)$$

$$U_\beta^{ref} = U_{sd}^{ref} \sin(\theta) + U_{sq}^{ref} \cos(\theta). \qquad (11)$$

These values can then be passed to the PWM generator, see Fig. 3.

Now, two different frameworks for motor modeling can be applied: the (d,q) co-ordinate and the (α,β) co-ordinate system. The dynamics of the BLDC motor in (d,q) co-ordinates is given by [3]:

$$L_{sq}\dot{I}_{sq} = -L_{sd} z_p \dot{\theta} I_{sd} - c_M \dot{\theta} - R_s I_{sq} + U_{sq} \qquad (12)$$

$$L_{sd}\dot{I}_{sd} = L_{sq} z_p \dot{\theta} I_{sq} - R_s I_{sd} + U_{sd} \qquad (13)$$

$$J_M \ddot{\theta} = -b_M \dot{\theta} + c_M I_{sq} - M_{load} \qquad (14)$$

where z_p denotes the number of pole pairs, Ψ_p the rotor flux and $c_M = z_p \cdot \Psi_p$ the machine constant, R_s, L_s the resistance and inductance respectively. J_M is the moment of inertia, b_M the internal friction and finally M_{load} the external load. Obviously, this is a non-linear differential equation with the two electrical states I_{sd}, I_{sq} and the one mechanical state $\dot{\theta}$. When neglecting the sd component (i.e. $I_{sd} = 0$), or, equivalently, starting off with a simple DC motor to model the BLDC motor's dynamics, we obtain (with appropriate initial conditions):

$$L_{sq}\dot{I}_{sq} = -R_s I_{sq} - c_M \dot{\theta} + U_{sq} \qquad (15)$$

$$J_M \ddot{\theta} = c_M I_{sq} - b_M \dot{\theta} - M_{load} \qquad (16)$$

The output signal is, in both cases, the current. Alternatively, the motor equation could be written down in (α, β) co-ordinates as follows:

$$U_\alpha = R_s I_\alpha + \frac{d}{dt}\left(L_s I_\alpha + \Psi_p \cos\theta\right) \qquad (17)$$

$$U_\beta = R_s I_\beta + \frac{d}{dt}\left(L_s I_\beta + \Psi_p \sin\theta\right) \qquad (18)$$

Since the models contain a lot of physical parameters plus some error terms, these have to be estimated and validated from data. The parameters then are estimated with e.g. prediction error methods, see [5] or unknown-but-bounded approaches, including parameter validation on validation data and a final assessment of the parameter quality. An overview and comparison of recent methods is given by [8].

STATE ESTIMATION

Luenberger Observer

Consider the linear equations (15,16) written down in standard state space notation and neglecting the load input, i.e.:

$$\dot{x}(t) = \begin{pmatrix} -\frac{R_s}{L_{sq}} & -\frac{c_M}{L_{sq}} \\ \frac{c_M}{J_m} & -\frac{b_m}{J_m} \end{pmatrix} x(t) + \begin{pmatrix} \frac{1}{L_{sq}} \\ 0 \end{pmatrix} U_{sq}; x(0) = x_0 \qquad (19)$$

$$y(t) = \begin{pmatrix} 1 & 0 \end{pmatrix} x(t) \qquad \text{with} \qquad x(t) = \begin{pmatrix} I_{sq}(t) \\ \dot{\theta}(t) \end{pmatrix} \qquad (20)$$

or shorter:

$$\dot{x}(t) = Ax(t) + Bu(t); \qquad x(0) = x_0 \qquad (21)$$

$$y(t) = Cx(t). \qquad (22)$$

Since this is an observable linear model, an estimate $\hat{x}(t)$ of the state vector $x(t)$ can be calculated using a Luenberger observer that measures output $y(t)$ and input $u(t)$:

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + K\left[y(t) - C\hat{x}(t)\right]. \qquad (23)$$

The error between real and estimated state vector $e(t) := x(t) - \hat{x}(t)$ tends to zero whenever the observer gain K is chosen properly, i.e. the system matrix $A - KC$ of the differential equation for the error

$$\dot{e}(t) = (A - KC)\, e(t); \qquad e(0) = e_0 \tag{24}$$

has eigenvalues in the left half plane. Based on the linear motor model in (d,q) co-ordinates (15,16), the Luenberger observer (23) is the first means to estimate the motor speed $\dot{\theta}(t)$ (and hence the position) based on measurements of currents and voltages.

Reduced Luenberger Observer

Since the current $I_{sq}(t)$ is state variable and (measured) output at the same time, it is not necessary to estimate it. Hence, the observer can be reduced to a one that only estimates the unknown state $\dot{\theta}(t)$. Therefore, the system is rewritten to (omitting the time argument t from now on):

$$\begin{pmatrix} \dot{y} \\ \dot{x_2} \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \begin{pmatrix} y \\ x_2 \end{pmatrix} + \begin{pmatrix} B_1 \\ B_2 \end{pmatrix} U_{sq} \tag{25}$$

$$y = \begin{pmatrix} 1 & 0 \end{pmatrix} x \tag{26}$$

The estimated value is x_2 and $x_1 = y$ is measured. Introducing new signals

$$u_r = A_{21} y + B_2 U_{sq} \tag{27}$$

$$y_r = \dot{y} - A_{11} y - B_1 U_{sq} \tag{28}$$

and new matrices $A_r = A_{22}, \quad B_r = 1, \quad C_r = A_{12}$, (25,26) become

$$\dot{x}_2 = A_r x_2 + B_r u_r \tag{29}$$

$$y_r = C_r x_2 \tag{30}$$

representing a system with a non-measurable state $x_2 = \dot{\theta}$. For this system, the Luenberger observer (as above) is:

$$\dot{\hat{x}}_2 = (A_r - K_r C_r)\, \hat{x}_2 + B_r u_r + K_r y_r \tag{31}$$

using K_r for the observer gain to be chosen. Resorting to the notation of (15,16) and introducing the new state $x_B = \dot{\hat{x}}_2 - K_r y$ yields the reduced observer for this case:

$$\dot{x}_B = \left(-\frac{b_m}{J_m} + \frac{c_M}{L_{sq}}\right) \hat{x}_B + \frac{K_r}{L_{sq}} U_{sq} + \left(\left(\frac{c_M}{L_{sq}} - \frac{b_m}{J_m}\right) K_r + \frac{c_M}{J_m} + \frac{z_p R_s}{L_{sq}}\right) I_{sq} \tag{32}$$

$$\dot{\hat{x}}_2 = x_B + K_r I_{sq}. \tag{33}$$

Based on the linear motor model in (d,q) co-ordinates (15,16), the reduced Luenberger observer (32,33) is the second means to estimate the motor speed $\dot{\theta}(t)$ (and hence the position) based on measurements of currents and voltages.

Extended Kalman Filter (EKF)

The Luenberger observer discussed above can be converted into a Kalman filter (and vice versa). The observer gain K would then be chosen with respect to the statistical properties of process noise and measuring error, which makes the Kalman Filter to an optimal state estimator in a statistical sense. The Extended Kalman Filter (EKF) is an extension of the (linear) Kalman Filter to handle non-linear systems. Hence, an extended Kalman Filter could be used to estimate the motor speed/position.

We consider the non-linear equations (12,13,14) in discrete time form and with process noise $v(k)$ and measuring error $w(k)$ (denoting k the sampling instance and T_s the sampling time):

$$x(k+1) = f(x(k), u(k)) + v(k); \qquad x(0) = x_0 \tag{34}$$

$$y(k) = g(x(k)) + w(k), \tag{35}$$

where $v(k)$ and $w(k)$ are uncorrelated sequences with the covariance matrices Q, R respectively:

$$E\{v(k) v^T(k)\} = Q \tag{36}$$

$$E\{w(k) w^T(k)\} = R. \tag{37}$$

The key idea of the EKF is to linearise the non-linear system at each time step and apply the machinery of the linear Kalman filter to this linearisation. The operating point is the state at the last sampling step. Hence, consider the following linearisation (denoted by $\Delta x(k) := \frac{x(k+1) - x(k)}{T_s}$ etc.) of equations (34,35):

$$\Delta x(k+1) = F(k) \Delta x(k) + G \Delta u(k) + v(k); x(0) = x_0 \tag{38}$$

$$\Delta y(k) = C \Delta x(k) + w(k) \tag{39}$$

with

$$F(k) := \left[\frac{\partial f}{\partial x}\right]_{x=x(k), u=u(k)} = \begin{pmatrix} \left(1 - \frac{T_s R_s}{L_{sd}}\right) & \frac{T_s z_p L_{sq}}{L_{sd}} \dot{\theta}(k) & \frac{T_s z_p L_{sq}}{L_{sd}} i_{sq}(k) \\ -\frac{T_s z_p L_{sd}}{L_{sq}} \dot{\theta}(k) & \left(1 - \frac{T_s R_s}{L_{sq}}\right) & -\frac{T_s z_p}{L_{sq}} \left(\psi_R + L_{sd} i_{sd}(k)\right) \\ \frac{T_s c_M}{J_m} & 0 & \left(1 - \frac{T_s b_m}{J_m}\right) \end{pmatrix} \tag{40}$$

$$G(k) := \left[\frac{\partial f}{\partial u}\right]_{x=x(k), u=u(k)}$$

$$C := \left[\frac{\partial g}{\partial x}\right]_{x=x(k), u=u(k)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}.$$

The EKF now derives the time update of the state at time $k+1$ given the knowledge at time k: $\hat{x}(k+1|k)$ using the non-linear system equation with measurement data of the input at time k:

$$\hat{x}(k+1|k) = f(\hat{x}(k|k), u(k)). \tag{41}$$

The quantity $\hat{x}(k|k)$ is the estimate of the state space vector is derived by using measurement data of the output at time k:

$$\hat{x}(k|k) = \hat{x}(k|k-1) + K(k)\left(y(k) - g(\hat{x}(k); u(k))\right). \tag{42}$$

Here, the filter gain $K(k)$ is then given by:

$$K(k) = P(k|k-1)C^T\left(CP(k|k-1)C^T + R\right)^{-1}. \tag{43}$$

Having this framework at hand and given initial choices for $P(1|0), x(1|0)$, the state update can be calculated. For the next iteration step, the estimated error covariance matrix is calculated as follows

$$P(k|k) = (I - K(k)C)\,P(k|k-1) \tag{44}$$

$$P(k+1|k) = F(k)P(k|k)F(k)^T + Q. \tag{45}$$

Nonlinear (α,β) Feedforward Filtering

Starting off with the motor equtions in (α,β) co-ordinates (17,18), and dividing one equation by the other, a simple relation can be obtained quite easily:

$$\theta = -\arctan\left(\frac{U_\alpha - R_s I_\alpha + L_s \frac{dI_{s\alpha}}{dt}}{U_\beta - R_s I_\beta + L_s \frac{dI_\beta}{dt}}\right). \tag{46}$$

Here we assume constant Ψ_p. Essentially, one does a (non-causal) dynamic filtering of the currents (differentiating) and a static one of the voltages, followed by a non-linear static filtering. Because the image of the $\arctan$ is limited to the interval $[-\pi/2, \pi/2]$ the absolute position of the rotor has to be derived by postprocessing using the information on $\sin, \cos$ and a counter.

Discussion of the Methods

The four methods presented so far qualify for estimation of the motor angle. We presented linear, as well as non-linear filters, observers and simple feedforward methods. Since we aim at using the algorithm in a production time of electronic control unit and not only on a rapid prototyping system, the computational load produced by the algorithm clearly is an issue. Table 1 gives an overview of the number of scalar multiplications, integrators and trigonometric functions unsed in the algorithm. Here, all algorithms are viewed as starting from (α,β) co-ordinates, which means that the Park transform is included in those methods, based on (d,q) co-ordinates. On the other hand, the feedforward method (46) estimates an angle in the interval $[-\pi/2, \pi/2]$, which has to be transformed into an absolute one. As can be seen in Table 1, the reduced Luenberger observer and the feedforward method produce a similar computational load, while the EKF is the most expensive one of the four.

Method	Multiplications	integrators	trigonometric and logic
Luenberger	19	3	4
reduced L.	24	2	4
EKF	115	13	4
Feedforward	7	2	6

Table 1 Computational load of the different algorithms.

The observer based methods guarantee convergence (even for wrong initial conditions), which is not the case for the feedforward method (46). In particular, the feedforward method could suffer from wrong initial condtions in the block that differentiates the current (which has numerical disadvantages as well).

RESULTS AND EXAMPLES

Figs. 4 and 5 show representative measurements from vehicles equipped with active front steering. The algorithms have been executed using a dSpace device, which has also been used for storing the data. Units for measuring motor voltages, currents and position are regular production line ones. As obvious from the measurements, the observer based methods follow the overall dynamics pretty well, whereas the feedfoward estimate drifts away from time to time. Since estimation error is not included in that method, the feedforward estimate usually keeps an offset. As visible in Fig. 6, the feedforward filter keeps better track particularly at higher speeds. A feature, inherent in the observer methods is reported in Fig. 7: since the motor's speed rather than its position is a state of the model, the observer could keep an offset as soon as the motor stands still. This could be resolved by including an extra state for the EKF.

The overall conclusion can be drawn that the observer based methods give tracking performance and accuracy as required by the safety concept. On the contrary, the feedforward filter particularly suffers from not feeding back the estimation error and hence drifting away from time to time.

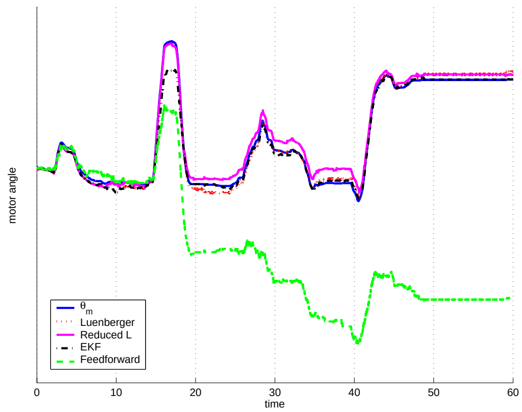

Figure 4: Measurements in a protoype vehicle showing measured motor angle (blue, solid), Luenberger observer (red, dotted), reduced Luenberger observer (magenta, solid), EKF (black, dash-dotted) and feedforward filtering (green, dashed).

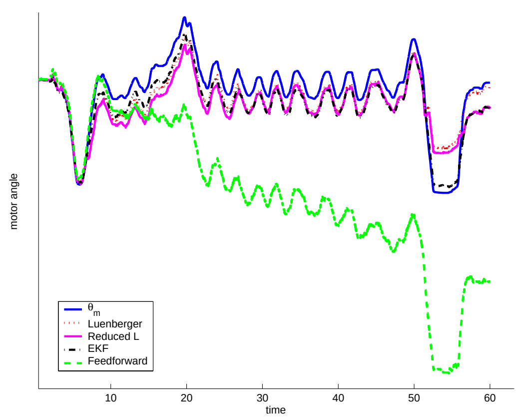

Figure 5: Measurements in a protoype vehicle showing measured motor angle (blue, solid), Luenberger observer (red, dotted), reduced Luenberger observer (magenta, solid), EKF (black, dash-dotted) and feedforward filtering (green, dashed).

CONCLUSIONS AND FUTURE WORK

Four method have been presented for estimating the motor position based on motor currents and voltages: a Luenberger observer, a reduced Luenberger obeserver, an Extended Kalman Filter and a non-linear feedforward filter. While the first three methods are observer based, the fourth

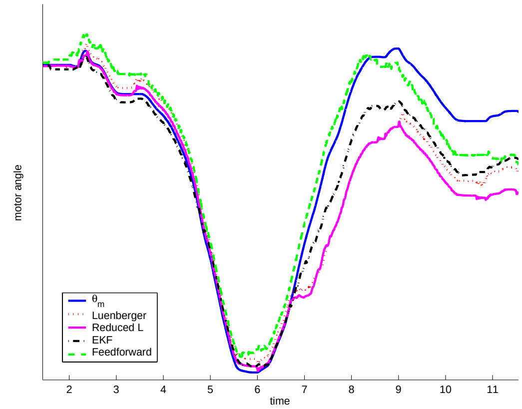

Figure 6: Zoomed version of Fig. 5 showing measured motor angle (blue, solid), Luenberger observer (red, dotted), reduced Luenberger observer (magenta, solid), EKF (black, dash-dotted) and feedforward filtering (green, dashed).

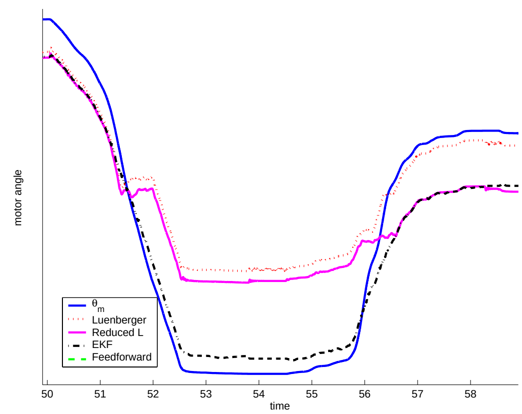

Figure 7: Zoomed version of Fig. 5 showing measured motor angle (blue, solid), Luenberger observer (red, dotted), reduced Luenberger observer (magenta, solid), EKF (black, dash-dotted) and feedforward filtering (green, dashed).

is not. Experiments in a vehicle show sufficient tracking performance and accuracy of the observers, but not of the feedforward filter. Differences in performance between the Luenberger observer and its state reduced version do not seem to be too significant (for our purpose).

Future work will concentrate on investigating temperature dependence of the model and running the two most promising options of the obeserver, namely the reduced state Luenberger obeserver and the Extended Kalman Filter on a production type electronic control unit using automated code generation.

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