IDENTIFICATION OF A FLEXIBLE BEAM

Wolfgang Reinelt[a] **and S. O. Reza Moheimani**[b]

[a] ZF Lenksysteme GmbH, 73522 Schwäbisch Gmünd, Germany.
Email: `Wolfgang.Reinelt@zf-lenksysteme.com`

[b] Department of Electrical and Computer Engineering, University of Newcastle, NSW 2308, Australia. Email: `reza@ee.newcastle.edu.au`

Abstract

A model for simply-supported flexible beam is identified from frequency data. The beam's displacement (invoked by applying a voltage to a piezoelectric transducer attached to the beam) is measured at 23 *different positions over the beam's surface. The resulting SIMO systems is identified by means of basis functions, for which the parameters are extracted directly from the data set and tuned afterwards. The mode shapes are obtained by spline interpolation.*

Keywords: Flexible structures; system identification; spatially distributed models; modal analysis.

1 Introduction and motivation

Flexible structures such as beams, plates, etc., are spatially distributed systems whose dynamics are governed by partial differential equations. For control design purposes these partial differential equations are approximated by lumped systems within a specific bandwidth. Given the spatially distributed nature of these systems their associated models describe spectral, as well as spatial characteristics of the system within a specific bandwidth.

For flexible structures such as beams, plates and strings with regular shapes and some specific boundary conditions, the task of discretizing the PDE into a lumped spatio-temporal model can be performed via the modal analysis procedure (Meirovitch, 1986). For a more complicated structure, however, modal analysis may not be a suitable approach as the task of finding an analytic solution to the resulting eigenvalue problem may prove too difficult. In such circumstances, a numerical method such as FEM may prove more effective.

System identification is a viable approach in modeling spatio-temporal systems, such as flexible structures, if one is able to capture the spatially distributed nature of the system in the model. Such models can then be used in designing spatial controllers, see e.g., (Halim & Moheimani, 2001). This paper is an attempt to demonstrate how a spatially distributed model for a flexible beam can be obtained. All experiments were performed in the Laboratory for Dynamics and Control of Smart Structures at the University of Newcastle, Australia. The experimental rig consists of a beam with experimentally pinned boundary conditions at its both ends. A piezoelectric transducer is

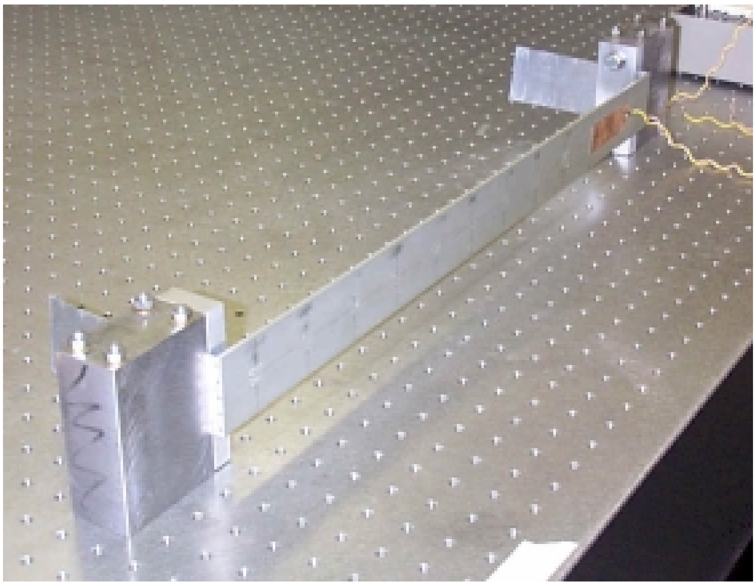

Figure 1: Experimental setup of the flexible beam, pinned at both ends.

attached to the beam and is used to vibrate the structure. A Laser Scanning Vibrometer is used to measure displacement on the surface of the beam at a number of positions. This data is then used to obtain a spatially distributed model of the system.

The paper is organized as follows: Sec. 2 describes the experimental setup for data acquisition, discusses the choice of the model class and presents the identification methods which we will be using. Having fixed the model class, Sec. 3 deals with the actual parameter estimation, which is further refined in Sec. 4. Based on the results obtained so far, the mode shapes are calculated in Sec. 5 and Sec. 6 gives a summary and gives directions for future works.

2 Data set, selection of model class and identification method

Data is collected from the experimental set up described in references (Moheimani, 2000) and (Halim & Moheimani, 2001). The experimental test-bed consists of a uniform aluminum bar of rectangular cross section. The beam is $60\ cm$ long, $5\ cm$ wide and $3\ mm$ thick. The beam is experimentally pinned at its two ends to generate boundary conditions close to the pinned boundary conditions at both ends. A pair of piezoelectric ceramic patches (PIC151) are attached symmetrically to either side of the beam, $5\ cm$ away from one of the pinned ends. In our experiments only one of the piezoelectric transducers is used to vibrate the beam.

The set up may be viewed as a single-input infinite-output (SIIO) system, where the input to the system is the voltage applied to the actuating piezoelectric transducer and the output is the displacement over the entire surface of the beam. Given that the width of the beam is considerably smaller than its length, within the bandwidth of interest, we may view the system as a one-dimensional spatially distributed system. That is all in-bandwidth vibration modes of the beam are assumed to be due to bending modes.

The beam displacement is measured at 23 different positions using the PSV300 laser scanning vibrometer. The resulting data can be regarded as data obtained from a SIMO system. We do know from physics that all the associated 23 SISO systems share the same poles. In our case, the first four modes are excited by the input signal, hence we have a parameterization by four second order resonant systems. This leads to the following model class:

$$G^j(s) = \sum_{k=1}^{4} \frac{\theta_k^j}{s^2 + 2\xi_k\omega_k s + \omega_k^2}, \quad j = 1, \dots, 23. \tag{1}$$

The input/output behavior from the voltage applied to the piezoelectric transducer u to the displacement x at position r^j on the beam is given by $x(r^j) = G^j \cdot u$. We use j as index for the position on the beam and k for the mode number. Identification task is now to calculate θ_k^j, the mode shapes, and the parameters ξ_k, ω_k. We will discuss the problem of calculating ξ_k, ω_k in the following two sections. Now, we assume, that a reasonable estimate for ξ_k, ω_k is available. Then (1) is linear in θ_k^j. We will employ two different methods to estimate θ_k^j. These are the least squares estimate

$$\theta_{k,LSE}^j = \arg \inf_{\theta \in \mathbb{R}^n} ||y - \sum_{k=1}^{4} B_k \theta_k^j u||_2. \tag{2}$$

and the restricted projection estimate:

$$\theta_{k,RPE}^j = \arg \inf_{\theta \in \mathbb{R}^n} ||y - \sum_{k=1}^{4} B_k \theta_k^j u||_\infty \tag{3}$$

(or an equivalent expression when having frequency data). Here, we have used the abbreviation $B_k = B_k(s; \xi_k, \omega_k) = \frac{1}{s^2+2\xi_k\omega_k s+\omega_k^2}$ for the basis functions. Note, that the only difference between the two estimates is due to the norm used. The least squares estimate (LSE) looks for the best estimate in the 2-norm, while the restricted projection estimate (RPE) uses the ∞ norm for this purpose. Both estimates can be applied to time- as well as to frequency domain data. Computation of the LSE involves a QR factorization of the information matrix (Ljung, 1999), while the RPE can be computed via linear programming (Garulli, Kacewicz, Vicino & Zappa, 2000). A comparison of both methods is given in (Akcay, Hjalmarsson & Ljung, 1996).

3 Choice of basis functions and identification of the mode shapes (SIMO case)

We will now discuss how to derive initial values for the basis parameters $(\omega_1, \xi_1), \dots,$ (ω_4, ξ_4), eight parameters in total. As the basis functions B_k are independent of the position on the beam at which the measurements were taken, and hence the parameters ω_k, ξ_k, we may obtain the basis function using one data record, say, the first one. First, we extract the four resonance frequencies ω_k as $125, 473, 1063, 1911 rad/s$ directly from the data record. For some choice of the ξ_k, we calculate an LSE and then tune the four parameters ξ_k, so that the resonance peaks of the measured (frequency domain) data and the amplitude plot of our estimate match exactly. It is noted that the four remaining parameters can be tuned quite independently from each other to accomplish this aim. This gives quite a good basis with continuous time poles located at

$$-0.1 + 125.66i; -0.5 + 473.66i; -2.8 + 1063.37i; -3 + 1911.78i.$$

We then run all 23 data sets with this basis expansion. Figures 2 and 3 show sample results. Additional investigations show, that these results can not be improved significantly by adding additional terms to the basis consisting of four second order elements in order to increase the degree of freedom when estimating the four complex zeros with just four real parameters. Another interesting observation is that LSE and RPE procedures effectively result in the same estimate, although they use different "philosophies" in the noise assumption: a soft energy bound versus a hard bound. Comparing the identified model to the analytical one, as derived in (Moheimani, 2000), it is difficult to say which one may be better than the other. In some cases, one method may estimate a zero better than the other, but it is difficult to state, that one method delivers an *in general* better result than the other. In all cases, the identified model matches the resonance top better then the analytical model (although this is no surprise at all, since this was the tuning criterion for the choice of the basis). An advantage of the identification procedure is that it is quick and fully automated, starting off with the measured frequency data, in contrast to the "by-hand" tuning of the parameters after physical modeling.

4 Tuning the basis parameters

We will now describe how to improve the quality of the basis parameters. This method is useful especially when having a second data record at hand, containing a different set of frequencies. We assume, that initial values for ω_k, ξ_k and an estimate of θ_k^j have been calculated as explained in the previous section. For simplicity of presentation, we restrict ourselves to a certain position on the beam (i.e. to one output).

In order to get an update of the basis function B_k in a simple way, we linearize the basis function B_k around the preliminary ω_k, ξ_k. Denoting the above setup as (the index j is omitted in this section):

$$\begin{aligned} G(s) &= \sum_{k=1}^{4} \frac{\theta_k}{s^2 + 2\xi_k\omega_k s + \omega_k^2} =: \theta^T \cdot h(s;\omega,\xi), \qquad (4) \\ &\omega := (\omega_1, \ldots, \omega_4)^T \\ &\xi := (\xi_1, \ldots, \xi_4)^T \\ &\theta := (\theta_1, \ldots, \theta_4)^T \\ &h(s;\omega,\xi) := (B_1(s;\xi_1,\omega_1), \ldots, B_4(s;\xi_4,\omega_4))^T. \end{aligned}$$

This leads to the following expression:

$$G(s;\Delta\omega,\Delta\xi) \doteq \theta^T \cdot h(s;\omega,\xi) + \theta^T \nabla h(s;\omega,\xi) \cdot \begin{pmatrix} \Delta\omega \\ \Delta\xi \end{pmatrix}. \qquad (5)$$

Note, that the linearization gives a bilinear parameterization in the parameters θ_k and $\Delta\omega, \Delta\xi$, i.e. keeping $\omega_k, \xi_k, \theta_k$ from the initial step constant and estimating $\Delta\omega, \Delta\xi$ is a *linear* problem and can be solved using (2) or (3). Having solved this problem, an update for the basis parameters is available as

$$\hat{\omega}_k = \omega + \Delta\omega, \hat{\xi}_k = \xi_k + \Delta\xi. \qquad (6)$$

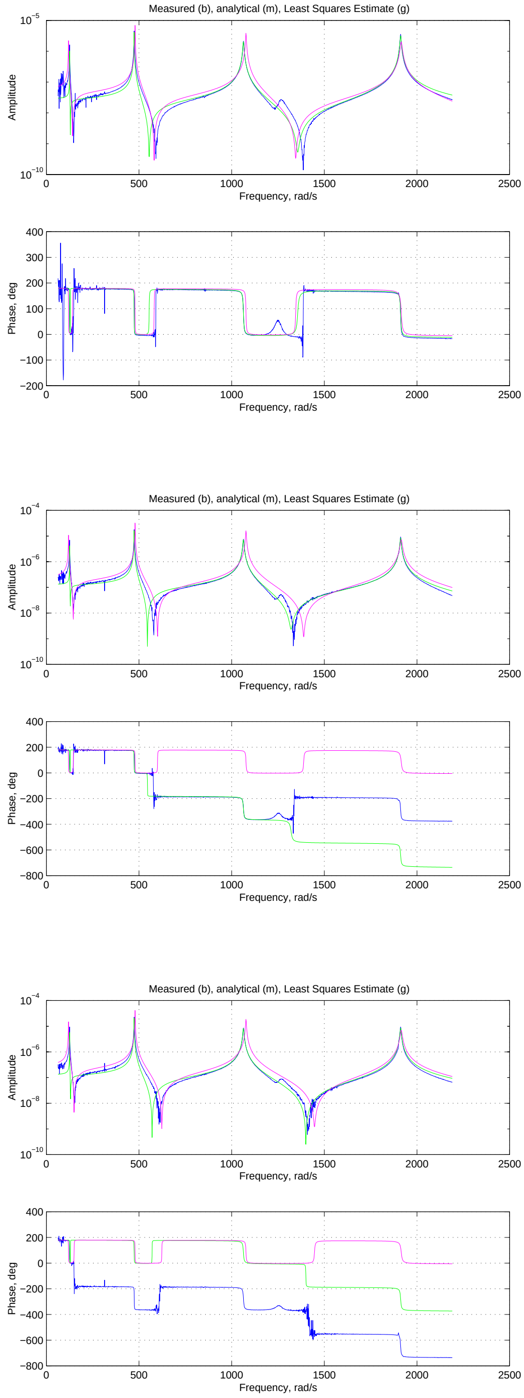

Figure 2: Identified LSE model (g), analytical model (m) and measured data (b) for outputs $j = 1, 3, 4$.

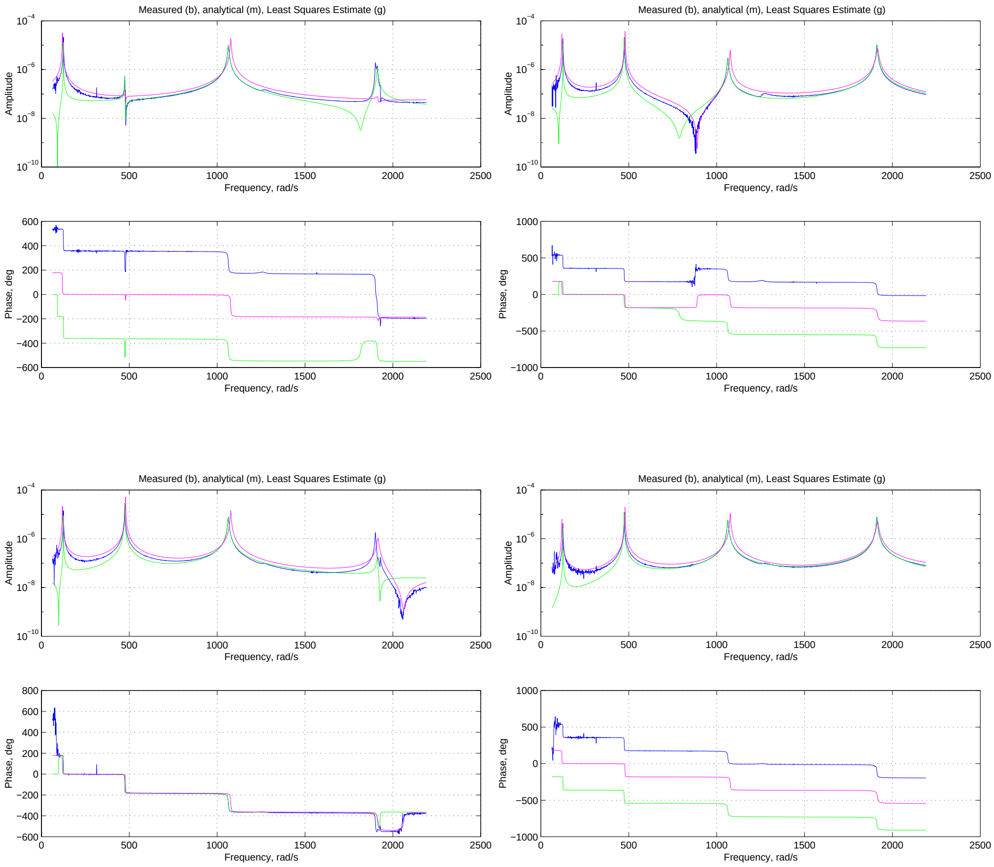

Figure 3: Identified LSE model (g), analytical model (m) and measured data (b) for outputs $j = 12, 15, 18, 22$.

Using the new basis that now depends on the improved $\hat{\omega}_k, \hat{\xi}_k$, we may estimate a new $\hat{\theta}_k$, and iterate this loop. This technique is known as boot-strapping for bilinear systems (Ljung, 1999, sec.10.4). Note, however, that this system is "artificially" bilinear due to the linearization applied in (5). We summarize the procedure shortly as:

1. Initial guess: extract resonance frequency ω_k and damping ξ_k from measured data.

2. Estimate the mode shape: θ_k via (2) or (3).

3. Carry out the linearization in (5) to obtain the bilinear setup:

$$G(s) \doteq \theta^T \cdot h(s;\omega,\xi) + \theta^T \nabla h(s;\omega,\xi) \cdot \begin{pmatrix} \Delta\omega \\ \Delta\xi \end{pmatrix},$$

   calculate $\Delta\omega, \Delta\xi$ using (2,3) and update $(\hat{\omega}, \hat{\xi})^T$ according to (6).

4. Replace ω, ξ by $\hat{\omega}, \hat{\xi}$ and go to step 2.

Moreover, this technique can be used for assessing the error associated with the parameters θ, ω_k, ξ_k by calculating the feasible ellipsoid when employing the restricted projection estimate RPE (3), for instance.

5 Calculation of the mode shapes (spatial case)

The final question that needs to be addressed is the following: we measured the displacement of the beam at different, distinct, locations, ending up in a SIMO system. Is it possible to calculate the displacement $x(r)$ for an arbitrary r, based on the model obtained in the above section? The answer is positive.

What we did above was to estimate the parameter θ_k^j (LSE or RPE), where k denotes the number of the mode $(1,\ldots,4)$ and j indicates the dependency on the location along the beam r_j. In order to obtain displacement at an arbitrary position r on the beam, we interpolate the four mode shapes $\theta_1^j,\ldots,\theta_4^j$ over location r_j using cubic splines and the proper boundary conditions. The result is reported in Figure 4. The mode shapes obtained here resemble mode shapes of a simply-supported beam (Meirovitch, 1986). That is, the first half of a sine-wave for the first mode, a complete sine wave for the second mode, and so on. Here, only every second point was used for interpolation, remainder of measured points are used as "validation" data. An alternative to this approach would be a parameterization of the mode shapes in terms of trigonometric functions as $\theta_k(r) = A_k \cdot \sin(\tau_k \cdot r)$ and to estimate (A_k, τ_k) from data.

6 Conclusions and future work

A spatially distributed system, a flexible beam, has been identified from measured data. The beam's displacement (invoked by applying a voltage to a piezoelectric transducer attached to the beam) is measured at a number of distinct positions along the beam. While the choice of the model class/structure is straightforward from physics, all parameters are determined from data. Possibilities to enhance an initial estimate of the parameters and to assess their uncertainties are discussed as well. The spatial behavior

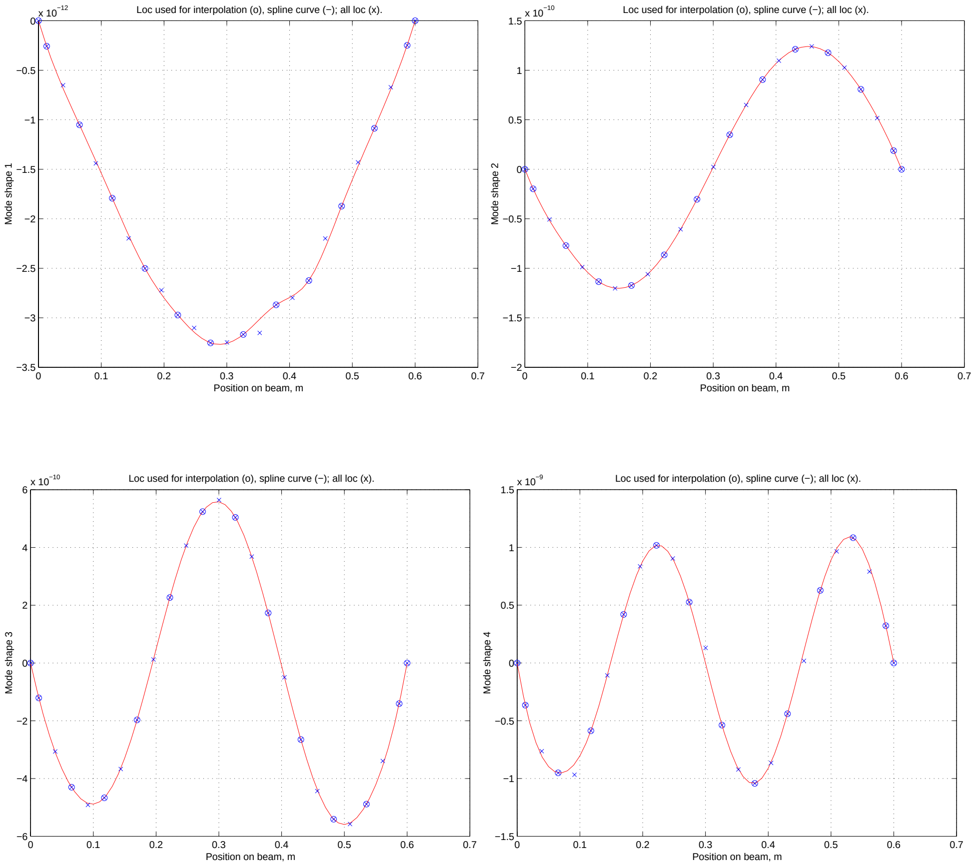

Figure 4: Four mode shapes $\theta_1(r), \ldots, \theta_4(r)$ depending on location on the beam. Data points used for interpolation (o) and validation points (x).

is characterized by the mode shape as a function of the of the position on the beam; it is determined by interpolation of the identified SIMO system.

In future works, a spatial controller design, directly based on this identified model will be considered and compared to existing ones. Another issue is the identification of two dimensional flexible structures, which is significantly easier when having a fully automated procedure for identification of the parameters.

Acknowledgments

The authors wish to thank Mr. Dunant Halim who performed the experiments and collected the data used in this paper. The work was carried out while the first author was with the Department of Electrical Engineering at Linköping University, Linköping, Sweden; therefore, financial support in part by the European Commission through the program Training and Mobility of Researchers - Research Networks and through the project System Identification (FMRX CT98 0206), as well as by the Swedish Research Council (Vetenskapsrådet) is gratefully acknowledged. Both authors acknowledge contacts with the participants in the European Research Network System Identification (ERNSI); in particular, valuable discussions with Lennart Ljung are gratefully acknowledged.

References

Akcay, H., Hjalmarsson, H. & Ljung, L. (1996), 'On the choice of norms in system identification', *IEEE Trans. on Automatic Control* **41**(9), 1367–1372.

Garulli, A., Kacewicz, B. Z., Vicino, A. & Zappa, G. (2000), 'Error bounds for conditional algorithms in restricted complexity set membership identification', *IEEE Trans. on Automatic Control* **45**(1), 160–164.

Halim, D. & Moheimani, S. O. R. (2001), Experiments in spatial H_∞ control of a piezoelectric laminate beam, *in* S. O. R. Moheimani, ed., 'Perspectives in Robust Control', Springer Verlag, Berlin, Germany.

Ljung, L. (1999), *System Identification – Theory For the User*, 2nd edn, Prentice Hall, Upper Saddle River, NJ, USA.

Meirovitch, L. (1986), *Elements of Vibration Analysis*, 2 edn, McGraw-Hill, Sydney.

Moheimani, S. O. R. (2000), 'Experimental verification of the corrected transfer function of a piezoelectric laminate beam', *IEEE Trans. on Control Systems Technology* **8**(4), 660–666.