Model (In-)Validation from a $\mathcal{H}_\infty$ and μ perspective

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Abstract

We give a short overview on methods of Model (In-)Validation, that fit to the robust control framework. The idea is that the mismatch between a measured datum and an expected datum is explained by a disturbance signal w *and* an error model Δ, representing unmodelled dynamics. The key question is if there exists a pair (w, Δ), sufficiently small, that can produce the measured datum. In particular, we view the different approaches by Smith et.al. and Poolla et.al., their numerical solution and given examples.

1 Problem Setup

The general setup for robust control is depicted in figure 1(a): a generalized plant P with the inputs control signal u and disturbance w and an error Δ, representing unmodelled dynamics. The plant P is given (modelling, identification) and we have a measured datum (u_{meas}, y_{meas}). The question is: Does the datum fit to the model? The main idea is that the (possible) mismatch betweeen a measured datum and the expected (output-)datum is explained by a disturbance signal w and an error model Δ. The relation between inputs and output is given by ULFT:

$$\begin{pmatrix} z \\ y \end{pmatrix} = \begin{pmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \end{pmatrix} \cdot \begin{pmatrix} v \\ w \\ u \end{pmatrix} =: P \cdot \begin{pmatrix} v \\ w \\ u \end{pmatrix} \tag{1}$$

$$\Rightarrow y = \left(P_{21}\Delta(I - P_{11}\Delta)^{-1}[P_{12}, P_{13}] + [P_{22}, P_{23}]\right) \cdot \begin{pmatrix} w \\ u \end{pmatrix} =: \mathcal{F}_U(P, \Delta) \cdot \begin{pmatrix} w \\ u \end{pmatrix} \tag{2}$$

Figure 1(b) shows a simplification of the general case: a weighted additive error. As a special case, it "disables" the feedback ($P_{11} = 0$) and fixes the location of the disturbance ($P_{12} = 0$). These two simplifications make the optimization problem (presented in the next section) convex. We state the Problem, treated in the following sections:

> **Problem-Definition:** Suppose the setup in figure 1(b). The (scaled) model P (plant and weights) is given, also given a measured datum (u_{meas}, y_{meas}).
> Do there exist $||w|| \leq 1$ and $||\Delta|| \leq 1$ so that eqn.(2) holds?

Scaling is possible by exploiting $\mathcal{F}_U(\gamma P, \Delta) = \gamma \mathcal{F}_U(P, \gamma\Delta)$.

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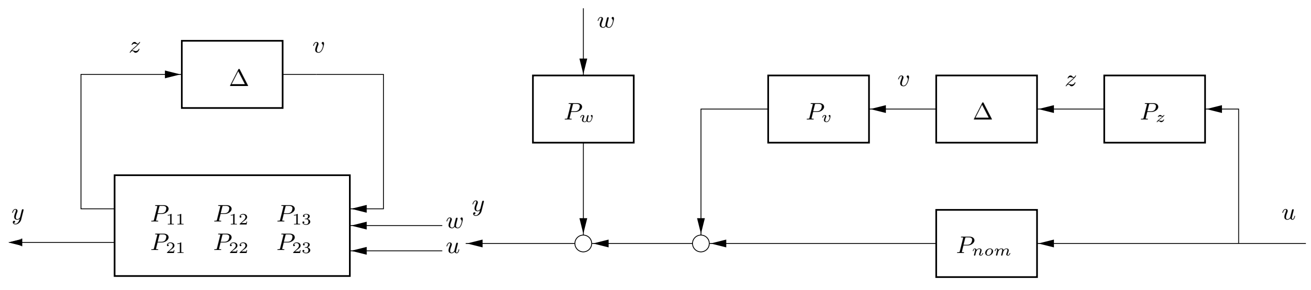

(a) General (U)LFT case: generalized plant P and error Δ.

(b) Simplified structure: weighted additive error Δ and nominal plant P_{nom}

Figure 1: Model validation framework.

2 Three Different Frameworks

We have a certain plant input u_{meas} with a measured output y_{meas}, a nominal output y_{nom} (without errors) and a modelled output y_{mod} (including the error model). The last two signals are calculated by the (error-)model with input u_{meas}. All signals are of finite length N. Ideally, the residuum $r = y_{meas} - y_{nom}$ should be zero[1].

To check the quality of the error model (w, Δ), we compare the measured output with the modelled output, i.e. we compare the residuum with $y_{mod} - y_{nom}$. Considering the above simplified plant structure, we get the following

$$\begin{aligned} r = y_{meas} - y_{nom} &\overset{?}{=} y_{mod} - y_{nom} \\ &= P_w w + P_{nom} u_{meas} + P_v \underbrace{\Delta P_z u_{meas}}_{=v} - P_{nom} u_{meas} \\ &= P_v v + P_w w \end{aligned} \tag{3}$$

Notable that there is no need fot P_{nom} to be linear. The lhs of eqn.(3) is known by measurement of y_{meas} resp. calculation of $y_{nom} = P_{nom} u_{meas}$. In the rhs, w varies by $||w|| \leq 1$ and v is given by

$$v = \Delta z \tag{$*$}$$

$$z = P_z u_{meas} \tag{5}$$

The rhs of eqn.(5) is known, eqn.($*$) contains the dynamics-error Δ, which will be removed in the next step. As eqn.($*$) must hold for all $\Delta \leq 1$, the question is, if there exists a relation between input z and output v? This question is answered by the so-called

> **Extension Theorem:** Eqn.($*$) holds for a $\Delta \leq 1$, iff the input signal z is larger than the output signal v.

Using this, eqn.($*$) degenerates to

$$v \text{ "smaller than" } z \tag{4}$$

[1]that's why the computational complexity of the problem depends in operator theoretical sense on the dimension of the kernel of the mapping $(v, w) \to r$, compare with eqn.(3).

and the two degree freedom optimization is reduced to a convex optimization problem: minimize $||w||$ with respect to (3,4,5). The result of the optimization is the minimum-norm disturbance w, responsible for the given datum (u_{meas}, y_{meas}). Finally, we get *sufficient* invalidation theorems of the following kind (dropping technical statements):

Invalidation Theorem: The model is invalid, if $||w|| > 1$.

The extension theorems for the three frameworks are responsible for the computaional complexity, because they increase the "size" of eqn.(4) in different amounts.

2.1 Discrete Frequency Domain (DFD)

Initial work: Smith [8], overview [1, sec 3], example [6].

The DFD approach transforms the time domain data, given in (3,4,5) into frequency domain data by DFT [1, eqns.(5-7)] for all N frequencies. The extension theorem for replacing the uncertainty Δ replaces eqn.(4) equivalently by

$$V_n^* V_n \leq Z_n^* Z_n, \quad \forall n \qquad \text{(DFD 4)}$$

The conditions for the optimization are eqns.(3,5) transformed into the frequency domain and eqn.(DFD 4). Exact formulation [1, Lemma 2 + Theorem 3].

Properties/Comments

- Quadratic objective + linear constraints $\Rightarrow$ no local minima.
- Even full sized LFT problems can be solved, applying μ techniques [8]. The problem remains convex as long as the SSV can be calculated by its upper bound (depends on the number of blocks).
- A computational example exists [6]: two liquids of different temperature are mixed in a tank (MIMO, 2×2), two different models are validated.
- Application of DFT: signals v, w have to be zero for negative times. No problem for v (depends on u), but for w, therefore restriction to static P_w, see [6, sec 2.3].
- Computational complexity $\sim N$ tractable.
- Another type of problem is posed in [8]: minimize the size of Δ and w. The problem is similar to the computation of μ, but not solved.

To avoid problems with DFT, we jump back into the time domain:

2.2 Discrete Time Domain (DTD)

Initial work: Poolla [7], overview [1, sec 4].

The DTD approach transforms the pulse response coefficients of P_v, P_u, P_z, given in (3+5) into their associated lower block Toepliz matrices (this a $N \times N$ matrix) and the signals into appropriate ones (N vector); see [1, eqns.(10+11)]. The extension theorem for replacing the uncertainty Δ replaces eqn.(4) equivalently by

$$V^* V \leq Z^* Z \qquad \text{(DTD 4)}$$

where V and Z are the associated lower block Toepliz matrices of the signals (i.e. eqn.(DTD 4) is a matrix inequality). Exact formulation [1, Theorem 4+5].

Properties/Comments

- Problem convex as long as Z^*Z constant ($\Leftrightarrow P_{11} = P_{12} = 0$). General case?
- No restrictions on v and w for negative times as in the DFD (appearing from DFT): nonzero v can be handled by residuum, nonzero w be initial conditions of P_w.
- Also LTV perturbations possible in the framework.
- This theory works also for multidimensional signals.
- Computational complexity $\sim N^5$, not feasible for reasonable data-length, even in LMI formulation [2, sec 4].
- Therefore no examples given

2.3 Sampled Data Domain (SDD)

Initial work: Smith and Dullerud [3] (technical details [5]), equivalent results independently derived by Poolla [4], overview [1, sec 5], example [2].

DTD regards the plant as a purely discrete-time system, with a "built-in" sampling time T. SDD is based on DTD, but interprets the plant as a sampled continuous system. The separation of plant and sample/hold unit enables us to *subsample*, i.e. we use the DTD-machinery for a subset of our data to get a feasible problem. The theoretical results are derived using the lifting operation. After this transformation, size and appearance of the extension theorem for replacing the uncertainty Δ are similar to the DTD case (including transformation to lower block Toepliz matrices):

$$\hat{V}^*\hat{V} \leq \hat{Z}^*\hat{Z} \qquad \text{(SDD 4)}$$

Exact formulation [1, Theorem 7+8].

Properties/Comments

- The invalidation theorem gets neccessary and sufficient for $T \to 0$ (which is only of theoretical interest).
- Same comments as in DTD, but subsampling possible. Start with subsampling time $T_{sub} \gg T$ and decrease until the model is invalid or $T_{sub} = T$.
- Example: heating system, SISO [2].
- Computational time within the example: model invalid for data-length of $N = 64$, this iteration-step needed $72 \cdot 10^3$ Mflops ($2h40mins$ CPU-time on Ultra1). The final step was the $6th$. [2, table 1].

3 Questions

1. DTD and SDD in case of full LFT: convexity is lost. Other solutions?
2. DFD solvable for full LFT problems (μ): implementation? examples?
3. Exploiting sparse structure in SDD to get a faster implementation [2]?
4. MIMO problems in SDD
5. "iff" invalidation theorems?
6. Suppose model is invalid because of $\min ||w||_2 = 1.36$, how to adjust the model knowing this value 1.36? Scaling and bounds in general?
7. All approaches compute the minimum size of w for all $||\Delta||_\infty \leq 1$. What about the question: (u_k, y_k) given, mimimum size of Δ and w [8, Problem 4.1]?

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(references in reversed chronological order)