ROBUST CONTROL OF SYSTEMS SUBJECT TO HARD CONSTRAINTS

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Preface

The presented dissertation is the outcome of three years of reseach done at the University of Paderborn (1995-1997).

I would like to thank my supervisor *Professor Dr. Felix Gausch* for having the opportunity to do my research in a rather privileged position as Research Assistant at the Institute for Automatic Control (University of Paderborn). Moreover, my thanks go to *Professor Dr. Anton Hofer* (TU Graz/Austria) for examinig my dissertation. I am mostly gratefull for several clarifying discussions and for the ample freedom they allowed me in conducting my research.

A number of people have contributed to the development of this dissertation. Especially, I would like to thank: *Mark Meierjohann*, *Georg Schwägerl* and *Günter Wegener* for excellent unix and network administration and *Ann Ernst* and *Mark Meierjohann* (again!) for carefully reading early versions of the dissertation.

Paderborn, February 1998 — Wolfgang Reinelt

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Chapter 1

Introduction

Motivation for Robust and Constraint Control

Most practical control problems are dominated by *constraints.* Valves can only be operated between fully open and fully closed, pumps and compressors have a finite throughput capacity and tanks can only hold a certain volume. Exceeding these prediscribed bounds causes unexpected behaviour of the system – large overshoots, low performance or (in the worst case) instability. A classic example for the detrimental effect of neglecting constraints is the Chernobyl unit 4 nuclear power plant disaster in 1986 [31].

Process models are always inaccurate – even extremely detailed models may contain unknown or changing physical parameters; so the controller has to manage the difference between the model (used for design) and the real plant. We motivate this by an anecdote, found in the works of H.S. Black [3]: "...every hour on the hour – twenty four hours a day – somebody had to adjust the filament current to its correct value. In doing this, they were permitted plus or minus 0.5 to $1dB$ variation in the amplifier gain, whereas, for my purpose the gain had to be absolutely perfect. In addition, every six hours it became necessary to adjust the battery voltage, because the amplifier gain would be out of hand. There where other complications too..." (Control-)Systems,

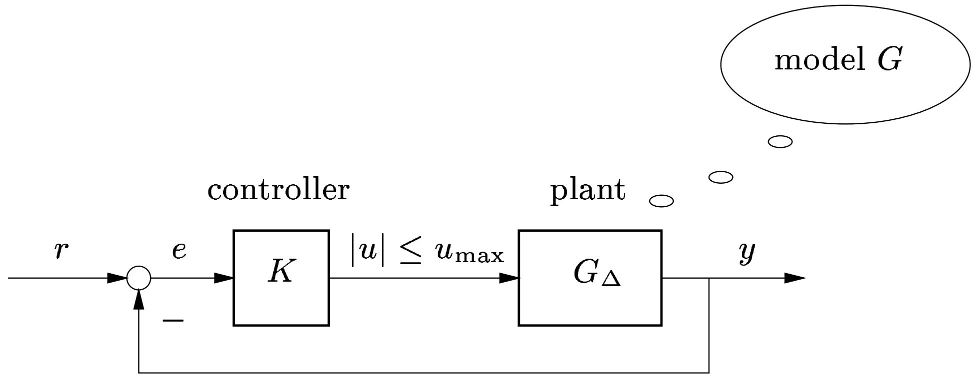

Figure 1.1: Control-loop with uncertain and constraint plant G_Δ.

that tolerate plant uncertainties or parameter variations are called *robust*. The system described above by Black is obviously not robust.

Either the effect of uncertainty or the effect of constraints is well-studied in control theory. Despite this, very little is known about robust control of systems with constraints. This field of control theory became interesting foremost in the last years.

Known Methods

Initial procedures for analysis and synthesis of robust controllers - qualitative methods [18] or quasiclassical methods of the "British school" [19, 24] – do not consider (hard) constraints directly. Early designs using $\mathcal{H}_\infty$ control theory check the amplitudes of the control variable later on by simulation [20]. Recent results on Linear Matrix Inequalities (LMI) allow additional LMIs to consider plant-input constraints [28].

Possibilities for the consideration of constraints are given by the concept of Anti Windup Bumpless Transfer (AWBT) or Model Predictive Control (MPC) [16, 21]. Both methods require intensive works for getting stability results of the closed loop. Initial works on AWBT and MPC neglect robustness, recent results add this feature. The application of MPC is restricted to slow dynamical systems (e.g. those appearing in process engineering).

A detailed overview of the methods stated above shows the lack of simple systematic methods to design robust, constraint and non-conservative controllers. This is carried out in the overview chapters.

MAIN RESULT

Within this thesis, we present a procedure for simple and systematic design of robust and constraint controllers. Therefore, the well known $\mathcal{H}_\infty$ Loop Shaping Design Procedure is extended so that the resulting controller meets the control variable's bound for a given set of reference signals. A strategy for the straightforward and systematic adjustment of the design weights after a loop shaping step is given. The presented procedure works for LTI multivariable (non-square) plants. Design examples illustrate the handling.

Thesis' Organization

All chapters of this thesis start with an introductory section named *Motivation and Overview.* Herein, the aims of the chapter are defined and the connection to the previous chapters is given. They end with a *Main Points and Conclusions* section, that reflects the chapter, collects open questions and links to the next chapter.

The first two chapters of this thesis are dedicated to a comprehensive overview of robust and constraint control:

Chapter 2: Robust Control Motivation for uncertain or inaccurate plants and aims of robust controller design are given. An overview of uncertain plant models and design methods is presented; this includes recent results as μ analysis, $\mathcal{H}_\infty$ control theory, Linear Matrix Inequalities and their applications in robust control.

Chapter 3: Constraint Control After a motivation, general results on stabilization of control-loops with bounded control variables are given. The common methods for analysis and synthesis of such control-loops are reviewed – including Anti Windup, Model Predictive Control and $\mathcal{H}_\infty$ methods. Robustness and performance are discussed. Finally, another set of admissible external signals (based on earlier works by Reichel) is discussed and its usage for controller design is pointed out. The numerical solution of the resulting optimization problem is discussed.

Within the overview on the state-of-the-art in robust control it becomes clear, that the $\mathcal{H}_\infty$ control theory is an extremly systematic and powerfull method for the design of robust controllers. Therefore, we discuss one $\mathcal{H}_\infty$ method more detailed – the $\mathcal{H}_\infty$ Loop Shaping Design Procedure:

Chapter 4: The $\mathcal{H}_\infty$ Loop Shaping Design Procedure This procedure, derived by McFarlane & Glover is reviewed in detail: basic stability results, uncertainty representation, robust stabilization and incorporation of performance properties are discussed.

These overview-chapters motivate the examinations in the second part of the thesis, because they show a lack of simple and systematic methods to design robust, constraint and non-conservative controllers. In the next chapters, we present a new approach for non-conservative and robust controller design subject to constraints:

Chapter 5: Extension of the $\mathcal{H}_\infty$ LSDP for Systems with Constraints The classical $\mathcal{H}_\infty$ Loop Shaping Design Procedure is extended for the consideration of control variable bounds. Reichel's ideas on admissible reference signals and maximum control variable are extended to the multivariable case. A strategy for the systematic adjustment of the design weights after a loop shaping step is presented.

Chapter 6: Design Examples Three design case studies show the usage of the prodedure in the SISO and in the multivariable case. Some necessary technical hints are collected.

We close the thesis with

Chapter 7: Conclusions The contributed results are reviewed and open questions are formulated. Suggestions for future works are given.

Basic Notation

The notation used in this thesis is standard from textbooks on optimal control theory. In the following, we state some basic definitions and abbreviations. All systems, we work on are supposed to be linear, finite-dimensional and time invariant.

1.1 Representation of Systems A continuous time state-space system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) + Du(t) \end{aligned}$$

will be denoted (A, B, C, D) or $\left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]$. The corresponding transfer-function is given by

$$G(s) = C(sI - A)^{-1}B + D$$

and will be abbreviated as $G(s) \stackrel{s}{=} (A, B, C, D)$ or $G(s) \stackrel{s}{=} \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]$. All transfer-functions are supposed to be real-rational. The space of all real-rational transfer-functions is denoted by $\mathcal{R}$. As "degree" of a transfer-function we will understand the "McMillan degree" [19, Definition 2.3].

1.2 Definition (∞-norm) The ∞-norm of a transfer-function matrix G is given by

$$||G||_\infty := \sup_\omega \overline{\sigma}\left(G(i\omega)\right)$$

where $\overline{\sigma}(\cdot)$ denotes the maximum singular value (sometimes, we denote the maximum singular value by $\sigma_{\max}(\cdot)$). $\mathcal{RL}_\infty$ denotes the space of all transfer-function matrices with finite ∞-norm, while $\mathcal{RH}_\infty$ denotes the space of all transfer-function matrices in $\mathcal{RL}_\infty$ with no poles in $Re(s) > 0$. Because we only work with real-rational transfer-function, we often write $\mathcal{H}_\infty$ instead of $\mathcal{RH}_\infty$.

Acronyms

In order to get a short notation, we use some acronyms that abbreviate often used or well-known terms:

ARE	Algebraic Riccati Equation
ARS	Admissible Reference Signal
ELSDP	Extended Loop Shaping Design Procedure
L(U)LFT	Lower (Upper) Linear Fractional Transformation
LHP	Left Half Plane
l(r)hs	left (right) hand side (of an equation)
LMI	Linear Matrix Inequality
LQG	Linear Quadratic Gaussian
LSDP	Loop Shaping Design Procedure
LTI	Linear Time Invariant
LTR	Loop Transfer Recovery
NLCF	Normalized Left Coprime Factorization
NPI	Nevanlinna-Pick Interpolation
MCV	Maximum Control Variable
MIMO	Multiple Input Multiple Output
MMP	Model Matching Problem
MPC	Model Predictive Control
RHP	Right Half Plane
SISO	Single Input Single Output
SSV	Structured Singular Value
QFT	Quantitative Feedback Theory

Cross-references, Citations and Subject Index

Parts of this thesis are cited e.g. as chapter 2 or section 3.6. For simplicity, subsections are cited as e.g. section 6.1.6, too. The first number within a figure- or table-reference refers to the chapter, e.g. figure 6.1, table 4.1. The same appears for equations, but they are always cited with brackets: eqn. (5.2) or equation (5.2).

The other organizing unit within this thesis is a label for Theorems, Definitions and other statements that need a marker. They are named e.g. "2.5 Definition" (boldface) but cited as definition 2.5 or (2.5), when "definition" is omitted. Again, the first number hints on the chapter. This marker does not distinguish between different names, so definition 3.1 is followed by remark 3.2 (and definition 3.2 does *not* exist). This is done for a better oversight.

External references are cited e.g. as [19], the references are resolved in the Bibliography, located at the end of this thesis (page 115). When we are in need of a more precise reference, e.g. [19, Section 2.2] refers to Section 2.2 of [19].

The Subject Index (page 119) contains terms that appear in this thesis and are either discussed in detail or have a precise (external) reference.

Chapter 7

Conclusions

7.1 Review of Contributions and their Relevance

Although a rich theory has been developed for robust control of linear systems *without* constraints, the field of robust control of linear system *with* constraints became interesting foremost in the last years.

This thesis focuses on the problem of controlling systems robustly in the face of hard constraints. The contributions in particular are (in order of their relevance):

Extension of the LSDP for Constrained Systems The main idea within this thesis is the extension of McFarlane & Glovers Loop Shaping Design Procedure (ELSDP) for constrained system, presented in section 5.2 (page 81). A short review is given in section 6.1 (page 87). The suggested method treats the design of robust and non-conservative controllers in the face of a hard bound on the control variable. It works for arbitrary LTI multivariable plants. Three details within this method are described next.

Extension of Reichel's Ideas (on ARS and MCV) to the Multivariable Case The works of Reichel were dedicated to non-conservative constrained controllers. They base on a certain description of the Admissible Reference Signals (ARS) and raise the question of the computation of the Maximum Control Variable (MCV). These aspects are extended to the multivariable case in section 5.1 (page 78). A strategy for the numerical computation is outlined.

Numerically Efficient Computation of the MCV The central algorithm for the computation of the MCV – the Balkenverfahren – was realized in a more effective way. This is mentioned in section 3.6.3 (page 58) and realized in [27].

Explicit Relation between MCV and Design Weights The key question within the presented ELSDP is the systematic adaption of the weights after a loop shaping step. Therefore, an explicit relation between MCV and design weights was derived in section 5.3 (page 83).

Detailed Design Examples Three detailed examples demonstrate not only the usage of the ELSDP, but also the different effect of the classical procedure. The examples are given in sections 6.2, 6.3 and 6.4 (page 89).

Comprehensive Overview of Robust or Constraint Control We started with an overview of state-of-the-art methods. Chapter 2 (page 9) reviewed several aspects of linear robust control: uncertainty models, limitations and design methods up to recent results in $\mathcal{H}_\infty$ optimization, μ-analysis and Linear Matix Inequalities.

Chapter 3 (page 41) focused on constraint control. Again, the used terminology and the examined problems were defined precisely and an overview of design methods and their limits was presented. Robustness was discussed in this context, too.

Chapter 4 (page 61) is a more detailed chapter on $\mathcal{H}_\infty$ Loop Shaping. It may serve as an overview of this method for those, who are interested in the cornerstones of this theory, but not on proofs or technical results (given in the original work).

These overview-chapters motivate the examinations in the second part of the thesis, because they show a lack of simple systematic methods to design robust, constraint and non-conservative controllers. These chapters are fitted with many cross-references to original works or major textbooks, so they may serve as introductory survey on robust or constraint control as well.

7.2 Open Questions on the ELSDP

Some details of the suggested procedure remain open. We state them here:

Meeting the Bound for the Uncertain Case Clear from the design objectives posed in section 6.1, the ELSDP does not solve the *robust* constraint control problem subject to the control variable. The desired bounds on the control variable only hold in the nominal case. Is an extension possible, so that the constraints hold in the face of plant errors, i.e. for the uncertain plant?

Hypothesis In example 6.2 (academic SISO example), we gave the following remark on suboptimal controllers: A suboptimal controller produces a "lower MCV $u_{\max}$" than the optimal one.

We extend this: Given two (suboptimal) controllers k_i with performance factors f_i and suppose $f_1 < f_2$ (k_1 is "better" than k_2), does $u_{2,\max} < u_{1,\max}$ hold?

This would be a usefull feature within the procedure. When a large stability margin was achieved, but the meeting of the bounds fails "slightly". Increasing the performance factor would decrease the Maximum Control Variable (and of course decrease the stability margin a little bit) and the problem is solved without new adjustment of the weights.

7.3 Suggestions for Future Works

Several issues, connected with the subject of this thesis and the suggested design method, are open:

Extension to other System Constraints Although the thesis' title contains the phrase " ... subject to hard constraints", we only focussed on control variable constraints. We did this because it is the most important hard bound, that appears in control engineering.

Despite this, the suggested method is quite easy to extend for other contraints than those on the control variable. The numerical computation is independent of the transfer function. During the weight adjustment, we will not get into difficulties, because the ∞-norm of the closed loop transfer function (examined when hard output bounds are desired) appears in another frequency range than the ∞-norm of T_{ru}.

Application of ARS and MCV in other robust controller designs We used the ideas of Admissible Reference Signals (ARS) and Maximum Control Variable (MCV) to extend the well-known $\mathcal{H}_\infty$ LSDP. These ideas can be added to other methods for robust controller design as well.

Discrete Time Systems Whenever designing $\mathcal{H}_\infty$-based controllers, we get high order controllers. So we must pose the question of realization. Designing the controller in the continious time-domain is one possibility that may be successfull. The more general strategy is a controller design in the q-domain. The main parts of the theory, presented within this thesis, will work in the discrete-time case as well: the $\mathcal{H}_\infty$ LSDP is applicable for plants in the q-domain; for the computation of the MCV in the z-domain, a numerical procedure exists. This is a field for some practical design case studies.

Comparative Studies It is clear, that the used set of Admissible Reference Signals produces non-conservative controllers in contrast to those controllers, that prerequisite $|r(t)| \leq R$. This has been worked out in several case studies in the non-robust case. Design examples comparing the ELSDP and e.g. an LMI-based method (theorem 3.12) can work out this phenomenon in the robust case.

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